**Q3(a)** We wish to show that

$$\int_C f \, \mathrm{d}s = \int_{-C} f \, \mathrm{d}s$$

Let $\boldsymbol{r}(t)$ be the parameterisation from $t = a$ to $t = b$. We reverse the curve by setting $u = -t$. Then we re-write the function $\boldsymbol{r}(-u) = \tilde{\boldsymbol{r}}(u)$. The key is to figure out the derivatives. Note

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\mathrm{d}u}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}u} = -\frac{\mathrm{d}}{\mathrm{d}u}$$

Then the LHS is

$$\begin{aligned}
\int_a^b f(\boldsymbol{r}(t)) \left|\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{r}(t)\right| \mathrm{d}t &= \int_{u=-a}^{-b} f(\boldsymbol{r}(-u)) \left|\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{r}(-u)\right| (-\,\mathrm{d}u) \quad \text{(make the } u = -t \text{ subst.)} \\
&= \int_{u=-b}^{-a} f(\tilde{\boldsymbol{r}}(u)) \left|-\frac{\mathrm{d}}{\mathrm{d}u}\tilde{\boldsymbol{r}}(u)\right| \mathrm{d}u \quad \text{(change the diff and relabel the function)} \\
&= \int_{-C} f \, \mathrm{d}s.
\end{aligned}$$

**Q3(b)** The work integral version is identical, except that there is no absolute value. So instead we have

$$\begin{aligned}
\int_C \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r} &= \int_a^b \boldsymbol{F}(\boldsymbol{r}(t)) \cdot \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{r}(t) \, \mathrm{d}t \\
&= \int_{u=-b}^{-a} \boldsymbol{F}(\tilde{\boldsymbol{r}}(u)) \cdot \left(-\frac{\mathrm{d}}{\mathrm{d}u}\tilde{\boldsymbol{r}}(u)\right) \mathrm{d}u \\
&= -\int_{-C} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r}.
\end{aligned}$$